# Quantitative Analysis of Metamerism for Multispectral Image Capture

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## Abstract

Surface metamerism is the phenomenon whereby two spectrally different reflectances induce the same response under fixed conditions (observer and illuminant) and induce different responses when recording conditions change. Metamerism arises due to the discrepancy between the degrees of freedom of the observer (usually three) and the degrees of freedom needed to represent surface spectral reflectances. In this paper we propose new ways to study the extent of this phenomenon.

We consider spectral images, model their induced response for one observer and illuminant, identify sets of surfaces that are spectrally different but induce the same response and subsequently examine these sets when changing the illuminant. If the surfaces under the new illuminant cease to induce identical responses, they are metameric reflectances and we further investigate the magnitude of the mismatch under the new illuminant by means of the CIE Lab  $\Delta E$  colour difference formula. Given this method we can determine the proportion of metameric surfaces within a scene as well as the degree to which metamerism in a scene is apparent.

#### Introduction

Trichromatic colour input devices, including the human visual system, coarsely sample the light  $E(\lambda)$ 

reflected from a surface  $S(\lambda)$ , forming the *colour* signal  $C(\lambda) = E(\lambda)S(\lambda)$ , through three broad filters. Colour formation thus encodes the continuous function of wavelength  $C(\lambda)$  as three values, roughly corresponding to the red (long wavelength), green (medium wavelength) and blue (short wavelength) power content, referred to as a tristimulus (CIE XYZ) in the human visual system or an RGB in trichromatic digital colour input devices (cameras, scanners, etc.). As a consequence of this encoding of the colour signal, there is a many-to-one relationship between physical surface descriptors (the surface spectral reflectance  $S(\lambda)$ ) and tristimuli (e.g. CIE XYZs). This relationship is known as *metamerism* [1, 2].

Metamerism is at the heart of colour image reproduction and in particular is the driving force of output imaging. Monitors, projectors as well as printers all work on the principle that it is sufficient to produce a *metameric match* instead of a *spectral match*, since the human visual system is unable to distinguish spectral differences due to the nature of colour image formation. As a consequence, spectra of surfaces reproduced on a monitor or in print can be far away from their original shape, yet look identical due to them being metameric matches.

Of course there are also disadvantages of this approach for output imaging. The metameric match achieved in reproduction is only valid so long as illuminant and observer don't change. If a match is achieved between original and reproduction under illuminant A, this match no longer needs to hold under illuminant B. From the point of view of colour input devices however, metamerism is a hurdle to overcome. Surface metamerism is both device and illuminant dependent: two surfaces that match for one observer under one light source, may become distinguishable for a second observer and/or under a second light source. Thus, if multiple, spectrally different surfaces induce identical XYZ tristimuli, there is no direct way to distinguish between these surfaces after acquisition. From the point of view of the input device, these surfaces have identical colour co-ordinates.

The problem of metamerism is being addressed by a host of approaches to multi-spectral imaging. There are several devices that capture more than three values at each spatial location. Assuming the channels are well designed (e.g. they are independent of each other and thus record data that is not redundant), the more channels there are in a device, the more accurately can surfaces be identified by their reflectances.

The design of multi-spectral device sensors goes hand in hand with the study of the nature of surface spectral reflectances. These have been found to be well modeled by statistical analysis, deriving lower-dimensional bases that represent them to a high accuracy, while reducing the original sampling. For example, the set of 426 Munsell colours [3] can be sufficiently well represented by a basis of about 6 principal components without significant loss in precision. This would suggest that if a 6-channel camera, having sensitivities equivalent to those of the six principal components of the Munsell colors, would capture a world only composed of surfaces statistically similar to the Munsells, then this camera would be able to uniquely capture each surface's spectral reflectance. However there are two principal problems with this approach. First and foremost, it is physically impossible to have sensors identical to principal components, as these have usually negative lobes. Second, it is difficult to have a representative set of surface reflectances that would represent all possible surfaces a device might encounter (and such set would be certainly more than 6 dimensional).

However, the argument is valid in that additional sensors help capture more variation in spectral reflectances and therefore improve the accuracy. One way to evaluate such multi-spectral or hyper-spectral devices is to examine directly their spectral reconstruction accuracy [4]. The results of such studies would yield an idea (in terms of CIE  $\Delta$ E's or RMS error) of how accurate the multi-channel capture is on a particular set of test surfaces. Another important aspect, directly linked to spectral reconstruction accuracy, is the level of metamerism that a device has.

If a device captures a scene such that the proportion of ambiguous responses is high (for many responses, more than one surface corresponds to a response), and therefore that the proportion of unique responses is low (few responses are in a 1 to 1 relationship with surface reflectances), then we say that its level of metamerism is high. Conversely, if a device is such that the proportion of ambiguous responses is low, and the proportion of unique responses is high, then we say that it's level of metamerism is low. A device with a low level of metamerism is a device with better reflectance estimation accuracy, whereas a device with a high level of metamerism is likely to perform poorly in reflectance estimation. A spectrophotometer for example has a level of metamerism of 0 as it records each surface as it's reflectance and thus all responses are unique.

In this study we wish to investigate how significant the phenomenon of metamerism is and how likely it is to have metameric surfaces in a scene, considering a variety of possible color capture devices, both trichromatic and multi-spectral. In our experiments we compare the CIE Standard Colorimetric Observer [5], a standard RGB digital camera and a 6 channel HDTV camera.[13]

In order to study different devices we need to be able to model the responses of the devices of scenes under different light sources. To have this freedom we will use sets of Multispectral images of both natural, outdoor scenes [6, 7] as well as images of common, man-made objects [8]. As each pixel in a multi-spectral image is a 31-dimensional wavelength vector, whereby the 31 dimensions correspond to a 10nm sampling between 400nm and 700nm, the images can be rendered under arbitrary illuminant spectral power distributions and arbitrary device spectral sensitivities following the laws of Mondrian-world colour image formation [9]. We consider the multi-spectral images as our ground-truth of surface spectral reflectances.

In our experiments we show the effect of the different types of colour capture device on the level of metamerism. We show that there is significant benefit from multiple recording channels from the point of view of metamerism, and therefore reflectance estimation. The HDTV device results, on average, in 6% surfaces being metameric, whereas the CIE XYZ observer 13% and the RGB camera in 11%.

Given a multispectral image with spectral reflectances  $S_i(\lambda)$ , their response vectors can be calculated via the following colour formation equations:

$$r^{x}_{i} = \int_{\omega} R_{x}(\lambda) E_{a}(\lambda) S_{i}(\lambda) \ d\lambda$$

where  $r^{x_i}$  is the response corresponding to the spectral sensitivity  $R_x(\lambda)$  of the *i*-th reflectance in an image and the integral is taken over the range  $\omega$  of visible wavelengths, in this case [400, 700] nm. Denoting  $\mathbf{R}(\lambda)$  the matrix of spectral sensitivity functions and  $\mathbf{r}_i$ a vector of device responses, we can re-write the above equation as:

$$\mathbf{r}_i = \int_{\omega} \mathbf{R}(\lambda) E_a(\lambda) S_i(\lambda) \, d\lambda$$

It can be seen from the colour formation equations in this equation, that there is a discrepancy between the response vector  $\mathbf{r}_i$  and the function of surface spectral reflectance  $S_i(\lambda)$  in that  $\mathbf{r}_i$  is a finite-dimensional discrete sampling of the infinite, continuous function  $S_i(\lambda)$ . Common colour input devices such as cameras, scanners as well as the human visual system, use three channels through which surface reflectance is integrated. For such devices the left hand side of the colour formation equation above is a  $3 \times 1$  vector. On the other hand, reflectances are commonly represented as vector quantities, usually resulting in a 31 x 1 vector on the right hand side. Thus there are three degrees of freedom on the left hand side, while there are 31 degrees of freedom on the right hand side. More generally, assuming a device with C channels and assuming reflectance is represented in a D dimensional vector space, the colour formation equations become a system of C linear equations of D unknowns. Since C << D, this system is under-determined and has D - C degrees of freedom (for C = 3 and D = 31 the system has 28 degrees of freedom) and it can be shown that for each **r** there is an infinite set of possible  $S(\lambda)$ 's which in colour science are referred to as metameric reflectances [2].

In order to overcome the problems introduced by metamerism into colour image reproduction, researchers have sought to devise methods to capture higher dimensional correlates of surface spectral reflectance. The most accurate approach is to capture the spectral reflectance itself by means of a spectrophotometer or telespectroradiometer, however both are costly and introduce further technical challenges. Consequently, intermediate approaches have been proposed whereby the number of capture channels C has been significantly increased from the three filters used in common devices to as many as 31 or more channels [4].

In this report however, we wish to study the degree to which the phenomenon of metamerism is found in nature. In particular we study multispectral images of natural and man-made objects and quantify the proportion of surface spectral reflectances in such scenes that are potentially metameric, given some change in illumination. Recently a study was conducted into the frequency of metamerism from the point of view of the human visual system's ability to identify materials, taking into account the mechanism of chromatic adaptation due to a change in the illuminant [10]. Our approach is new in that we look at not only the human visual system but also other digital colour input devices with three (RGB) and more (multispectral) spectral sensitivities. Considering general colour input devices, we shall be able to draw conclusions as to the extent of metamerism and it's impact on issues such as colour correction, device characterisation and chromatic adaptation.

Our results show that given an image of a natural scene, a significant proportion of its reflectances are spectrally different and induce the same response, however a much smaller proportion of these also exhibit the effect of metamerism under a change of illumination. We examined the worst-case colour mismatch of these metameric sets of reflectances under a large set of possible illuminants and found that in terms of CIE Lab  $\Delta E$  colour difference the worst-case mismatch is on average around 10  $\Delta E$  units. The illuminants for which such worst-case mismatch can occur are generally spectral and chromatic opposites to the canonical illuminant, as expected.

The article is organised as follows. First we define the framework and methodology by which we study the frequency of metamerism and we propose a number of measures and analyses of the extent of metamerism. Then we report results of applying the proposed analysis to three devices, two trichromatic and one multispectral. Finally we discuss the results and conclude our study.

### Experimental

In order to study the frequency of metamerism in multi-spectral scenes we find all sub-sets of reflectances from a multispectral image that are metameric. So, for a set of reflectances  $S_l(\lambda)$  and their corresponding responses  $\mathbf{r}_m$  for a given device with spectral sensitivities  $\mathbf{R}(\lambda)$  under illuminant  $E_a(\lambda)$ , we define the set  $\mathbf{P}_{[Ea, rm]}$ , such that a reflectances  $S_k(\lambda)$  is it's member if and only if:

For all 
$$S_l(\lambda)$$
 in  $P_{[Ea, rm]}$ :  
 $|S_k(\lambda) - S_l(\lambda)| > \varepsilon_S$   
and  
 $|\int_{\omega} \mathbf{R}(\lambda) E_a(\lambda) S_k(\lambda) d\lambda - \mathbf{r}_m| < \varepsilon_r$ 

whereby  $\varepsilon_S$  is a threshold value determining that reflectances  $S_k(\lambda)$  are different from all other reflectances  $S_l(\lambda)$  already in the set  $P_{[Ea, rm]}$  and  $\varepsilon_r$  is a threshold determining that the response of reflectance  $S_k(\lambda)$  is sufficiently similar to the chosen response  $\mathbf{r}_m$ . Due to metamerism it is clear that the number of sets  $P_{[Ea, rm]}$  has to be  $\leq$  the number of all reflectances in a scene. Their number is equal if all reflectances are spectrally distinct and at the same time induce different responses. In practice this is unlikely and most responses of an image will have corresponding sets with more than one member.

The set  $P_{[Ea, rm]}$  is thus the set of all spectrally different surface reflectances in an image that, within some set threshold  $\varepsilon_{S}$ , induce identical response under a particular illuminant, within some threshold  $\varepsilon_r$ . The reflectances in this set however, need not be metameric according to the definition used earlier. Each of the sets  $P_{[Ea, rm]}$  has to be examined under a change of illuminant or device/observer in order to establish whether the match under  $E_a(\lambda)$  is maintained, in which case the effect of metamerism is absent, or if this match breaks, in which case metamerism is exhibited.

Let us assume a set of illuminant spectral power distributions  $E_i(\lambda)$  representative of possible light sources. In our study we use a set of 173 spectral power distributions covering CIE standard illuminants (daylight simulators, incandescents as well as fluorescents) as well as measured actual light sources (natural and artificial) [11]. Figure 1 shows the spectral power distributions, normalised to have a maximum of 1, demonstrating large spectral variation. Figure 2 instead shows their corresponding CIE u'v' chromaticities with each illuminant plotted as an sRGB rendering of a perfect white diffuser under that illuminant, demonstrating the broad coverage of chromaticities.

In order to examine each of the sets of reflectances that induce identical responses under a chosen canonical illuminant  $E_a(\lambda)$ , we define the set  $Q_{[Ea, rm]}$  $\subseteq P_{[Ea, rm]}$  of reflectances  $S_k(\lambda)$  as follows:

$$\exists E_i(\lambda) \& \exists S_i(\lambda) \in \boldsymbol{P}_{[Ea, rm]}:$$
$$|\mathbf{r}_k^i - \mathbf{r}_l^i| > \varepsilon_r$$

where  $\mathbf{r}_{k}^{i}$  and  $\mathbf{r}_{1}^{i}$  are responses of reflectances  $S_{k}(\lambda)$ and  $S_{l}(\lambda)$  respectively under illuminant  $E_{i}(\lambda)$  and  $\varepsilon_{r}$  is the same threshold value determining whether two responses are significantly different.

If a set  $Q_{[Ea, rm]}$  is non-empty then the reflectances within are strictly metameric, otherwise the reflectances in the set  $P_{[Ea, rm]}$  are simply spectrally different reflectances that always induce the same response, considering the domain of possible change of conditions, the set of 173 test illuminants.



Figure 1: Illuminant spectral power distributions of 173 lights, normalised to have a maximum value of 1.



Figure 2: CIE u'v' chromaticity diagram of the 173 illuminants with the colors plotted corresponding to an sRGB rendering of the respective white points of each illuminant

We perform two analyses of the sets  $P_{[Ea, rm]}$  and  $Q_{[Ea, rm]}$ . Given a multispectral image, we express the proportion of reflectances from the image that are metameric (the number of unique reflectances that induce identical response divided by the total number of unique reflectances). This is a quantitative

expression of the extent of metamerism. Next we look at the extent of the metameric mismatch, by finding those conditions for which the reflectances result in a colour mismatch. This means finding at least one of the test illuminants for which the unique surfaces that record to identical response under canonical illumination cease to induce identical response and therefore result in a colour mismatch. This is an expression of the magnitude of metamerism in a given image.

So as to quantify the mismatch due to metamerism, we then examine the effect of all illuminants on each set  $Q_{[Ea, rm]}$  and find all illuminants  $E_i(\lambda)$  that cause a non-zero CIE Lab  $\Delta E$  colour difference among the reflectances in  $Q_{[Ea, rm]}$ . The CIE Lab  $\Delta E$  statistics then express an approximation of the perceptual magnitude of metamerism in any given image.

#### Results

First we describe the experimental set-up used in our simulations. Two sets of multi-spectral images were used in the experiments reported here. The first set contains 16 images from two Nascimento et al. data sets [6, 7] comprising predominantly of outdoor, landscape scenes containing buildings, woods, flowers, meadows, etc.. Both of these sets were captured with a monochromatic digital camera through a filter-wheel of 33 narrow band filters with peaks from 400nm to 720nm resulting in 33 dimensional vectors are each pixel location. Each image also included a sphere painted with a neutral grey, used to estimate the illuminant spectral power distribution in the scene. Using this estimate the images were processed to discount the effect of the illuminant and provide reflectance images. The second set contains the 24 images of the UEA Multi-spectral Image database [8] captured in a VeriVide viewing booth. The images contain everyday products and packaging as well as standard calibration charts such as the Macbeth ColorChecker Chart, the Macbeth Digital ColorChecker Chart, the Agfa IT8 Chart and the Esser Chart. Each image was captured twice under a fluorescent daylight simmulator of CIE illuminant D75, once with a white calibration tile, and once without. The white calibration tile was used to extract

the illuminant spectral power distribution in order to get surface spectral reflectance data. These images were captured using the Applied Spectral Imaging SpectraCube device that is based on the principle of interferometry. The spectral precision of this device which can be selected by the user, ranges from 300nm up to 1100nm and is non-uniformly distributed in the wavelength domain with a higher frequency in the short wavelength range and a lower frequency in the long wavelength rage. The capture precision used for the UEA Multi-spectral Image Database was set such that the maximum sampling interval was under 10nm (in the long wavelength range).

Both data sets were re-sampled to a uniform sampling interval in wavelength domain of 10nm steps between 400nm to 700nm resulting in 31 samples. This precision is common in spectral representation and has been found to be sufficient for smooth surfaces and illuminants [12].

We use three sets of sensor spectral sensitivities, plotted in Figure 4. First we report previous results on the CIE 1931 Standard Colorimetric Observer [5], the XYZ curves for comparison, then we compare them to a standard RGB digital camera (SONY DXC-930) and finally we show results for the 6 channel, High-Definition Television camera sensors designed by the Akasaka Natural Vision Project [13].

As the canonical illuminant we chose CIE illuminant D65, an off-white daylight. This is the illuminant we use in the first step of analysing metamerism. If a set of spectrally non-identical surfaces has identical response (XYZ, RGB or 6 band HDTV response) under this illuminant, we consider them as potential metamers (members of a set  $P_{[Ea, rm]}$ ) as they are ambiguous to the device. For test illuminants we use a set of 173 standard as well as measured illuminant spectral power distributions spanning a large part of chromaticity space. Both the spectra and the u'v'

chromaticities are plotted in Figures 1 and 2. These test illuminants are used in the second stage of the evaluation in order to establish if potential metamers (from the first stage) become distinct responses under any one of the 173 illuminants. These can then be considered actual metamers (members of a set  $Q_{[Ea, rm]}$ ).

Next we define the precision to which two reflectances or responses are considered equivalent, the repeatability threshold denoted  $\varepsilon_s$  and  $\varepsilon_r$ . Two reflectances are considered different if they differ in at least one sampled wavelength when quantised to an integer scale of [0, 100], and two responses are considered different if they differ in at least one of the recording channels when quantised to an integer scale of [0, 100].



Figure 3: Example images from the two sets of multi-spectral scenes rendered as sRGBs under CIE illuminant D65. Top six scenes are from the Nascimento et al. set and the bottom 11 are from the UEA set.



Figure 4: The CIE 1931 colour matching functions (top), the SONY DXC-930 RGB digital camera (middle) and the spectral sensitivities of the 6 band HDTV camera (bottom)

We report experiments in terms of the three measures described in the previous section. First we show the average statistics of the proportion of potential metamers in the scenes. This is the proportion of surfaces in a scene that record to identical responses under the canonical light source CIE D65, while having different spectral reflectances. Next we report statistics of average metamer proportions. This is the proportion of reflectances that are identical under the canonical light source (D65) while having different responses under any of the possible 173 test illuminants. Finally we quantify the degree of this mismatch in terms of CIE Lab  $\Delta E$  statistics.

Table 1 shows the summary statistics of the 40 multi-spectral images for each of the three devices in turn, CIE XYZ, SONY RGB and 6 band HDTV.

	min	μ	med	max	-
Pot. Metamers (%) Metamers (%) CIE Lab ⊿E	21 2 0.4	62 13 7.8	63 14 5.9	100 33 94.2	
SONY RGB Pot. Metamers (%) Metamers (%) CIE Lab ⊿E	29 1 0.3	68 11 6.1	69 12 4.8	100 30 94.2	
6 band HDTV Pot. Metamers (%) Metamers (%) CIE Lab ⊿E	1 0 0.4	43 6 8.0	38 5 6.5	100 20 91.1	

Table 1: Summary statistics of potential metamers, actual metamers and CIE  $\Delta E$  colour mismatch errors for 40 images under 173 lights.

First we look at the results of the potential metamers proportion (PMP). From Table 1 it can be seen that the CIE XYZ curves and the SONY RGB sensitivities have similar PMP for this set of images. The range for these two cases is between 62% and 69% of all unique surface reflectances. This means that between 62% and 69% of surfaces in a scene have an ambiguous (one to many) relationship between responses and reflectances. Conversely, it means that (100 - 69 =)31% to (100 - 62 =) 38% of surfaces in the scenes are in a one-to-one relation with responses. These are the surface reflectances that, using these devices, are likely to result in small estimation error. In comparison, the HDTV device has much lower PMP around 38%, consequently up to 62% of surfaces are uniquely identifiable from the response and are thus unambiguous.

A similar behavior can be seen in the reported statistics of the actual metamer proportions (MP), however these are significantly lower than the PMP. For the two tri-chromatic devices the % of surfaces that exhibit metamerism under any one of the 173 illuminants is between 11% and 14% on average, while for the 6 band HDTV this is between 5% to 6%.

This means that of the surfaces that potentially exhibit metamerism (PMP), only a fraction will actually break the match under D65. Thus 87%, 89% and 95% of surfaces respectively for XYZ, RGB and HDTV, are unaffected by metamerism given our experimental set-up.

Finally, let us consider the magnitude of the potential mismatch that the metameric surfaces could exhibit under any of the 173 test illuminants. The minimum, mean, median and maximum CIE Lab  $\Delta E$  statistics in Table 1 correspond to a CIE Lab colour difference between two reflectances that have equivalent response under CIE illuminant D65, while having different spectral reflectances and different responses under another one of the 173 illuminants. Each case of a mismatch is taken into account, so if a pair of reflectances is metameric under all 173 test lights then 173 colour differences will be considered. In this way the statistics express scenarios under any possible change in illumination.

The differences between the three devices are diminished in terms of this error metric. Both the two trichromatic and the 6 band device result, on average, in an error difference of between 4.8 and 8 units of  $\Delta E$ with a maximum of over 90  $\Delta E$ . A mismatch of 1  $\Delta E$ unit is designed to correspond to a just noticeable difference. Therefore, the above colour differences are perceptually significant. However, the CIE  $\Delta E$  colour difference was designed to express colour mismatch for uniform, coloured patches placed one next to another, whereas the above results may corresponds to two pixels of variable distance from each other in an image of 820 x 820 pixels. In order to express the perceptual extent of this mismatch, another metric would be necessary that takes both spatial location (are the two surfaces close by or far away?) as well as spatial extent (are the surfaces corresponding to large areas of uniform colour?).

The fact that there is no significant difference between the two trichromatic devices and the 6 band HDTV device is because additional sensors help reduce the ambiguity, but in the presence of ambiguity, its magnitude is not reduced. In other words, the six-channel device significantly reduces the proportion of surfaces that are metameric, however the surfaces that remain metameric cause the same magnitude of colour mismatch. Further investigation is needed to establish how the shape and number of spectral sensitivities affect the metamer proportions and the magnitude of the  $\Delta E$  mismatch.

## Conclusions

In this paper we extended the analysis of surface metamerism by examining three devices, the CIE Colour matching functions, an RGB camera and a 6 band HDTV camera. We conducted experiments on these three devices, modeling the responses to surfaces from 40 multi-spectral images of both natural and man-made objects, considering a change in illumination from D65 to 173 different spectral power distributions covering a large chromaticity area and representing standard as well as measured daylights, fluorescents and incandescents.

We have been able to quantitatively express the magnitude of the phenomenon of metamerism for each of the devices and therefore compare them from this point of view. Metamerism is directly linked to reflectance estimation in that the more metameric surfaces there are in a scene, the more difficult is it to estimate reflectances from the ambiguous responses. Our results have shown that an increase in the number of capture channels significantly reduces the proportion of metameric surfaces in a scene, but that the magnitude of the potential colour mismatch remains the same as for trichromatic devices.

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